[2] Pavel Pudlak, *Ramsey's theorem in bounded arithmetic*, ***4th Workshop, computer science logic***, Lecture notes in computer science, vol. 533, Springer-Verlag, Berlin Heidelberg, 1991, pp. 308–317.

▸ PAULO OLIVA, *On the extraction of polynomial-time algorithms from ineffective proofs in feasible analysis*.

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Consider as a *base for feasible analysis* Cook and Urquhart's [1] system $\mathsf{CPV}^{\omega}$ extended with the axiom of arithmetical choice for quantifier-free formulas $\mathsf{QF\text{-}AC}$. This system has a simple functional interpretation (via negative translation) in $\mathsf{IPV}^{\omega}$, which allows one to extract polynomial-time computable realizers from proofs of $\Pi^0_2$-theorems (i.e., formulas of the kind $\forall x \exists y A(x, y)$, $A$ being quantifier-free) in $\mathsf{CPV}^{\omega} + \mathsf{QF\text{-}AC}$. In [3] an extension of this functional interpretation to include a 'feasible' formulation of weak König's lemma for $\Pi^0_1$-definable trees is given by extending $\mathsf{IPV}^{\omega}$ with a simple form of binary bar recursion $\mathcal{B}$, i.e., it is shown that the theory $\mathsf{CPV}^{\omega} + \mathsf{QF\text{-}AC} + \Pi^0_1\text{-}\mathsf{WKL}^{\omega}$ has a functional interpretation (via negative translation) in $\mathsf{IPV}^{\omega} + \mathcal{B}$. Moreover, in [3] it is also proven that the type one terms of the system $\mathsf{IPV}^{\omega} + \mathcal{B}$ are polynomial-time computable. This gives a procedure for extracting polynomial-time realizers from proofs of $\Pi^0_2$-theorems in $\mathsf{CPV}^{\omega} + \mathsf{QF\text{-}AC} + \Pi^0_1\text{-}\mathsf{WKL}^{\omega}$. The $\Pi^0_2$-theorems are even allowed to makes use of 0-1 oracles as free-variables.

The base system $\mathsf{CPV}^{\omega} + \mathsf{QF\text{-}AC}$, which when extended with $\Pi^0_1\text{-}\mathsf{WKL}^{\omega}$ is strong enough for proving e.g., the Heine/Borel covering lemma, is an extension to higher types of a subsystem of Ferreira's [2] BTFA (Base Theory for Feasible Analysis), for which $\Pi^0_2$-conservation of WKL has been non-constructively proven.

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[1] S. Cook and A. Urquhart, *Functional interpretations of feasibly constructive arithmetic*, ***Annals of Pure and Applied Logic***, vol. 63 (1993), pp. 103–200.

[2] F. Ferreira, *A feasible theory for analysis*, ***The Journal of Symbolic Logic***, vol. 59 (1994), no. 3, pp. 1001–1011.

[3] P. Oliva, *Polynomial-time algorithms from ineffective proofs*, Submitted, 2003.

▸ MAURICIO OSORIO AND JUAN ANTONIO NAVARRO, *Modal Logic $S5_2$ and FOUR*.

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Our connectives for $S5$ are b (Box), $\sim$ (negation) and $\vee$ (or). $A \to B$, $A \ \& \ B$, and $d\ A$ are considered as abbreviations in the standard way.

Consider the axiom schemata:

$$F1 := d\ A \to b\ A,$$

$$F2 := ((d\ A1 \ \& \ d\ A2) \ \& \ b{\sim}(A1 \ \& \ A2)) \to b(A1 \vee A2), \qquad \text{and so on.}$$

For every Natural number $i$ the reader can picture the axiom schema $F_i$.

For every Natural number $i$, $S5_i$ denotes $S5$ extended with the schema $F_i$.

We show that $S5_i$ is determined by the class of reflexive, transitive, and symmetric frames of size at most $i$. $S5_1$ is the "not very interesting" (sometimes) called Trivial modal logic. This family of logics define a strictly descending chain, where $S5_i$ is strictly stronger than

$S5_{(i+1)}$. Moreover, we present a translation of $S5_2$ into the well known bilattice FOUR that preserves theorems. FOUR has the for values $\{0, 1, t, f\}$. We write & 1 and $\vee 1$ to denote the meet and joint operators w.r.t. truth ordering. We have also two negation operators $-1$ (negation) and $-2$ (conflation):

$$-1(t) := f, \quad -1(f) := t, \quad -1(1) := 1, \quad \text{and} \quad -1(0) := 0.$$
$$-2(t) := t, \quad -2(f) := f, \quad -2(1) := 0, \quad \text{and} \quad -2(0) := 1.$$

We define the function $f$ from modal formulas into FOUR formulas as follows:

$$f(A) := A \text{ if } A \text{ is atomic.}$$
$$f(b\,A) := f(A) \quad \&\,1 \quad -f(A).$$
$$f(\sim A) := -1\ -2\ f(A).$$
$$f(A \vee B) := f(A) \vee 1\, f(B).$$

A $t$-tautology is a formula that evaluates always to $t$. We show that $A$ is theorem of $S5_2$ iff $f(A)$ is a $t$-tautology.

We observe that $S5_2$ agrees with $S5$ in a non trivial fragment of formulas, interestingly enough for applications in computer science. Hence, we show that a small but relatively interesting fragment of $S5$ can be translated to FOUR.

► WIM RUITENBURG, *Intuitionistic theories whose Kripke models are all T-normal ones.*
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Given a set of sentences T, a Kripke model is T-normal, or locally T, if all its node structures are (classical) models of T. Samuel Buss constructs a theory H(T) which is sound and complete for the locally T Kripke models. We show that if, additionally, all models of H(T) are locally T, then H(T) is axiomatizable by geometric sentences. This answers a question implied by Wolfgang Burr's Mathematical Review MR 2001h:03115.

[1] SAMUEL R. BUSS, *Intuitionistic validity in T-normal Kripke structures*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 159–173.

► KSENIJA SIMIC, *Reverse mathematics of Hilbert spaces.*
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To date, little research has been done in the area of Hilbert spaces from the point of view of weak subsystems of second order arithmetic. In this talk, I will give the definitions and introduce basic concepts relevant to developing the theory of Hilbert spaces in this framework, and then state some results regarding orthogonal projections and closed sets. I will also discuss the mean ergodic theorem, and show that it is equivalent to arithmetic comprehension over the base theory $RCA_0$.

► STEPHEN M. WALK, *Blanketing functions and a conjecture about array computable degrees.*
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We define a *blanketing function* for a c.e. degree $\boldsymbol{a}$ to be a function $f \leq_{wtt} K$ that dominates all $\boldsymbol{a}$-computable functions. The concept comes from Downey, Jockusch, and Stob's definition of *array computable* [1]: A degree $\boldsymbol{a}$ is array computable, by definition, if and only if such a function $f$ exists.

Every blanketing function must have high c.e. degree, and it is natural to wonder *which*