# Consistent neg-extensions of superintuicionistic theories

Mauricio Osorio, Juan Antonio Navarro and Jose Arrazola

Universidad de las Américas, CENTIA, Sta. Catarina Mártir,
Cholula, Puebla, 72820 México
{josorio, ma108907}@mail.udlap.mx

## 1 Abstract

We consider only finite propositional theories defined over a signature $\mathcal{L}$. We use $I$ to denote intuitionistic logic and $G_3$ to denote Gödel 3-valued logic. A si-logic is any logic stronger or equal to $I$ logic and weaker or equal to $G_3$. A consistent neg-extension of a si-logic theory $T$, is a consistent extension of $T$ obtained by adding only negated formulas (i.e. formulas of the form $\neg\alpha$). Of particular interest is the case when these extensions are also complete. For a given theory $T$, we write $MS(T)$ to denote the set of such extended theories that are complete, under a background si-logic. Consider $T := \{\neg a \rightarrow b, \neg\neg c \rightarrow c\}$. Then $T$ has exactly two consistent and complete neg-extended theories, that is $MS(T) = \{\{\neg a, b, c, \ldots\}, \{\neg a, b, \neg c, \ldots\}\}$.

We present several interesting and useful results about such extensions that have important impact in logic programming and non-monotonic reasoning. Some of them are the following.

We have proved that for a given set $T$, $MS(T)$ is invariant with respect to the background si-logic. We define $T_1 \equiv_{MS} T_2$, iff for every theory $T$, $MS(T_1 \cup T) = MS(T_2 \cup T)$. We have shown that $T_1 \equiv_{MS} T_2$ iff $T_1 \equiv_{G_3} T_2$. We say that $MS(T)$ is c-minimal if the set of atoms in each neg-extended theory in $MS(T)$ is a minimal model (in classical logic). We have found some sufficient conditions on $T$, to ensure that $MS(T)$ is c-minimal. Our ultimate goal is to show that si-logics based on our approach provides a solid framework to model non-monotonic reasoning.

**Acknowledgements** This research is sponsored by the Mexican National Council of Science and Technology, CONACyT (project 35804-A). Our papers on this line of research are available online at http://www.udlap.mx/∼ma108907.